Exploring the Transformations of Graphs – Introduction

Mathematical Content:	Technical TI-Nspire Skills:
Functions	Drawing Graphs
Transformations of Graphs	Use of Sliders
Composite Transformations	Use of Functions

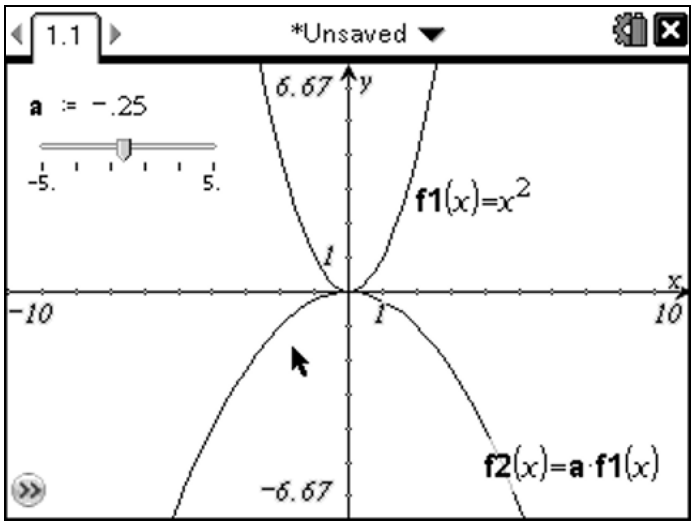
This activity uses the dynamic features of the TI-Nspire to look at four transformations. We create an environment in which the value of a constant **a** is controlled by means of a slider. We create an original function, $f1(x)=x^2$ and define four other functions in terms of f and **a**:

$f(x+a)$, $f(x)+a$, $f(a\cdot x)$ $a\cdot f(x)$

These second functions can be defined in terms of the first by writing them as, for example, $f2(x)=f1(x+a)$. Then, by adjusting the value of **a** using the slider, it is possible to explore the impact that **a** has on the transformation.

It is possible to explore all of the transformations outlined above by simply changing the function $f2$.

You can change the original function to be anything else by simply changing the function entered for $f1$, and the transformation will automatically change as well.



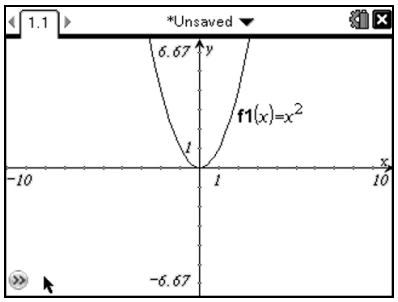
You can go on to extend the idea by including a second slider and then explore the effect of composite functions!

Exploring the Transformations of Graphs – Student Worksheet

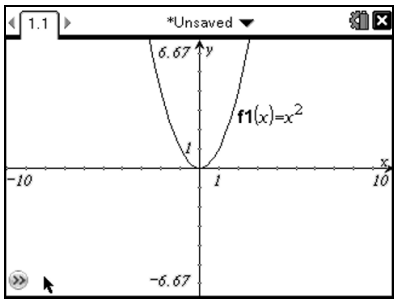
In this task you will explore how transformations of graphs can be expressed using function notation.

Task 1: *f(x + a)*

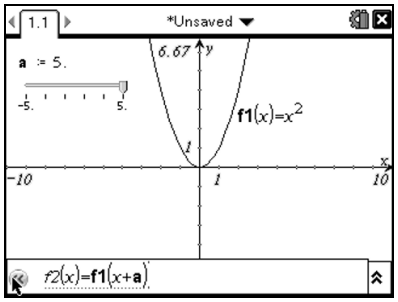
Start by creating a new document and on a Graphs page draw the graph of x^2. You could use any function here and at the end we will come back and change this.



Now add a slider to the page and change the variable on the slider to be **a**.

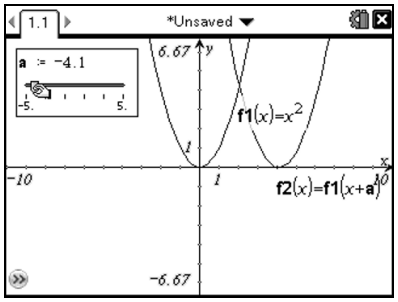


Change the settings for the slider so that it takes values between -5 and 5.



Now enter a second equation of the form *f2(x)=f1(x + a)*.

Explore the effect that changing the value of **a** has upon the graph by adjusting the value of the slider.

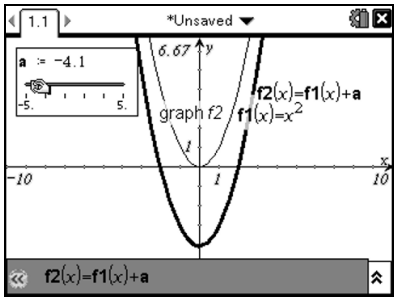


Describe the effect of the transformation *f(x + 3)*

Task 2: *f(x) + a*

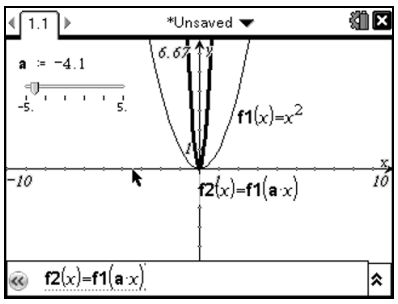
We can re-use our construction to explore other transformations. Bring the input line back up and change *f2* so that it represents the transformation *f2(x)=f1(x) + a*.

Again explore the effect that changing the value of **a** has upon the graph by adjusting the value of the slider



Describe the effect of the transformation *f(x) – 2*

Task 3: *f(a·x)*



Bring the input line back up and change *f2* so that it represents the transformation *f2(x)=f1(a·x)*.

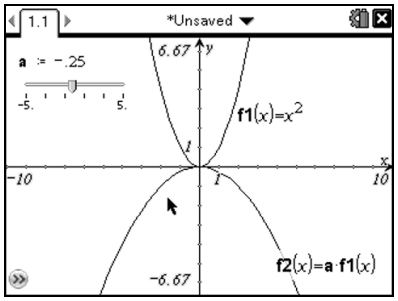
Note: You must press ⊗ between **a** and **x,** otherwise it will think you are talking about a variable called **ax.**

Again, explore the effect that changing the value of **a** has upon the graph, by adjusting the value of the slider.

Describe the effect of the transformation *f(2x)*.

Task 4: *a·f(x)*

Re-use your construction to look at *a·f(x)*. Bring the input line back up and change *f2* so that it represents the transformation *f2(x)=a·f1(x)*



Note: You must press ⊗ between **a** and **f(x)**, otherwise it will think you are talking about a function called *af(x)*.

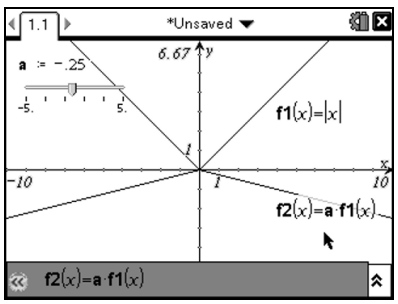
Again explore the effect that changing the value of **a** has in this context. What happens when **a** is negative?

Describe the effect of the transformation *4·f(x)*.

Extension

Because your transformed function *f2* is defined in terms of *f1* you can look at a different function by just changing the function entered for *f1*. Explore the effect that **a** has upon different functions.

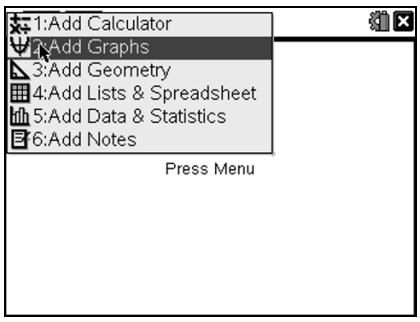
For example, on the right *f1(x)=|x|* has been used.



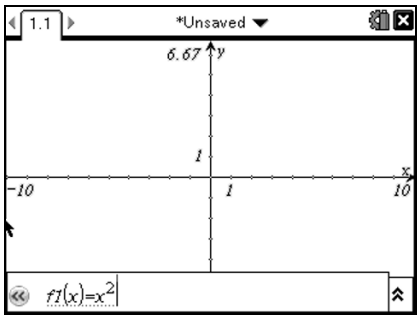
Exploring the Transformations of Graphs
Detailed Technical Help Sheet

Task 1

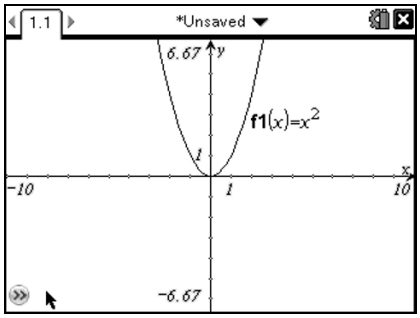
Step 1: Create a new document and then select option '2 – Add Graphs'.



Step 2: Here we create our primary function, x^2, although you will see later how to easily change this to explore a whole range of functions. For the x^2 function just press ⊠ (x²). Then press (enter).

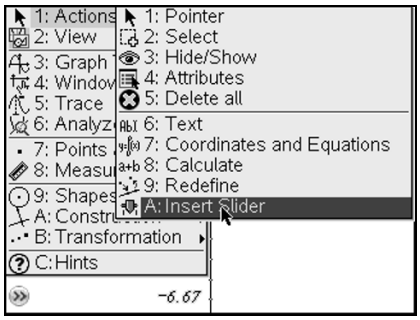


Step 3: Notice how the input line disappears allowing the full graphing screen to be seen.

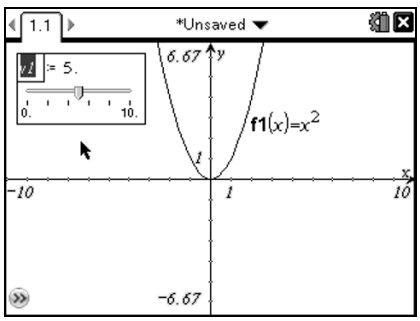


Step 4: Next add a slider to drive the transformation.

To add a slider press (menu), then choose option "1: Actions" and then option "A: Insert Slider".

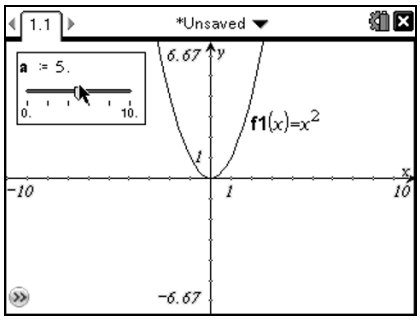


Step 5: The slider appears in the top right corner of the screen and the cursor flashes in the variable box containing **v1.**



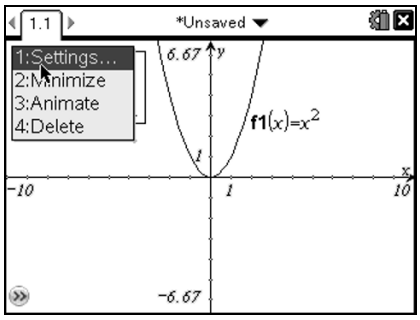
Step 6:

The variable can be whatever we want but for this example make it **a**.
To do this press (del) to delete the old variable name and then press A (enter).



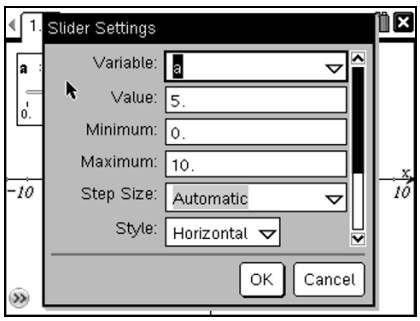
Step 7:

Currently the slider moves between 0 and 10, but we want it to run between -5 and 5. Change it by moving the cursor over the slider, pressing (ctrl)(menu), and selecting "1: Settings".



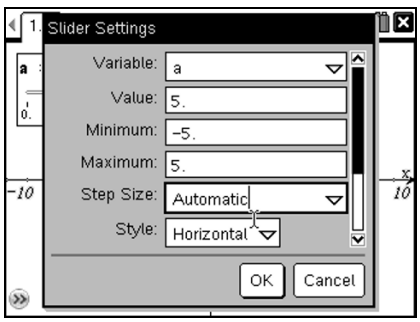
Step 8:

This settings wizard appears allowing you to change many of the properties of the slider.



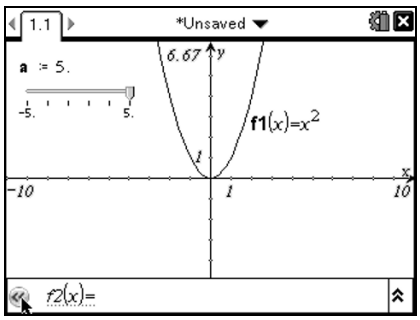
Step 9:

Change the minimum value to '-5' and the maximum value to '5'. If we wanted to the slider to go up in steps of one we would type 1 into the step size field, but we want it to be nice and smooth so will leave it saying "Automatic".

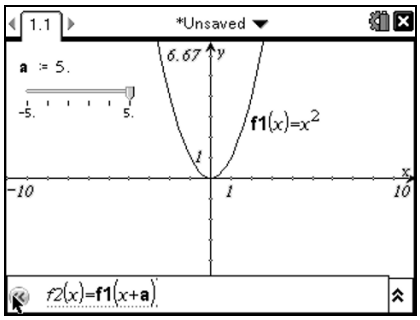


Step 10:

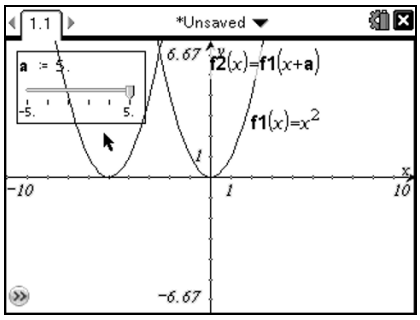
Now we want to draw another function based on our original function but driven by the slider. To do this we need to bring the input line back at the bottom of the screen, so press (ctrl)G or click on the double-arrow icon.



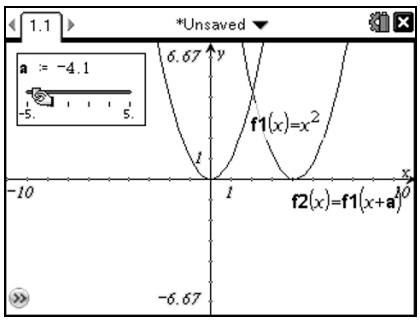
Step 11: We want to draw the function *f(x + a)*, so enter a new function based on the original one: *f2(x)=f1(x + a)*. Do this by pressing: F 1 (X + A) enter.



Step 12: The graph of the new function appears on the screen. To transform this graph move your cursor over the handle of the slider: to grab it, either press and hold ⬚ or press ctrl ⬚.

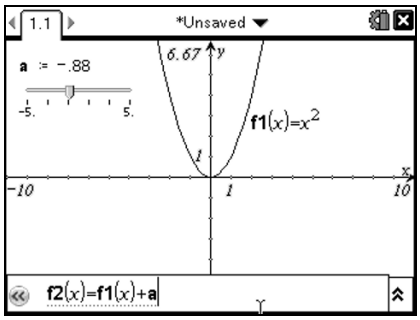


Step 13: Now use the Touchpad to adjust the value of the slider and observe the result on the screen.

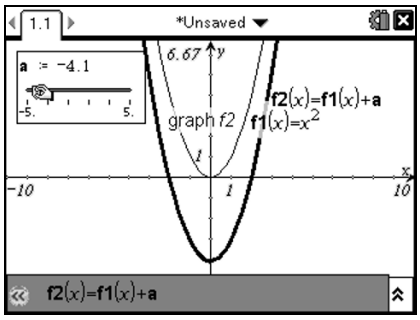


Task 2

Step 1: We can explore different transformations using the construction *f(x) + a.*
Press ctrl G to get the input line back on the screen, press ▲ to display the previous function and then use del to remove it. Replace it with *f1(a·x)* by pressing F 1 (X) + A enter.



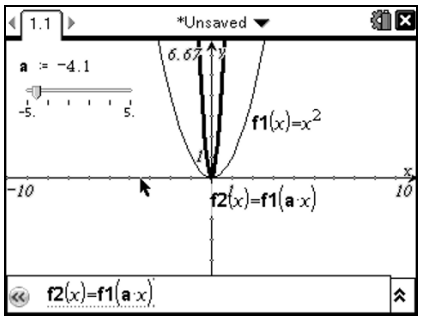
Step 2: Again grab the slider by moving the cursor over it and either press and hold ⬚ or press ctrl ⬚. Explore the effect of the value of **a** on the graph by dragging the slider around.



Task 3

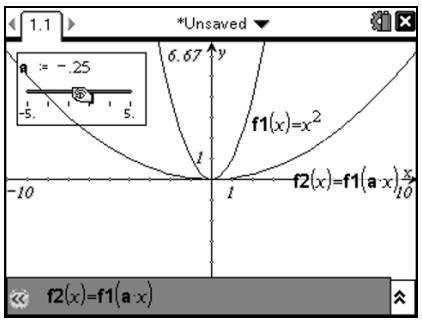
Step 1:

Now explore different transformations using the function *f(a·x)*.
Press ctrl G to get the input line back on the screen, press ▲ to display the previous function and then replace it with *f1(a·x)* by pressing F 1 (A × X) enter .
Note the need to press ×, otherwise it will assume **ax** is a variable.



Step 2:

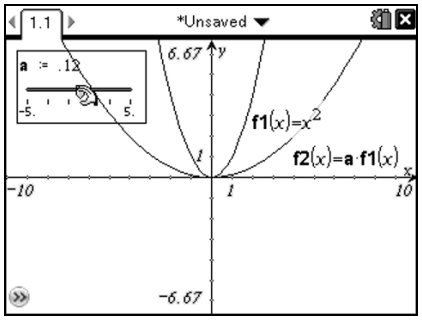
Again grab the slider and explore the effect of the value of **a** on the graph by dragging the slider around.



Task 4

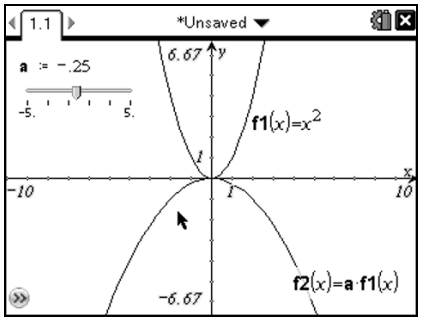
Step 1:

Now explore the transformation *a·f(x)*.
Press ctrl G to get the input line back on the screen and replace the previous function with *a·f1(x)* by pressing A × F 1 (X) enter.
Remember to press × or it will assume you mean *af(x)* is a new function.



Step 2:

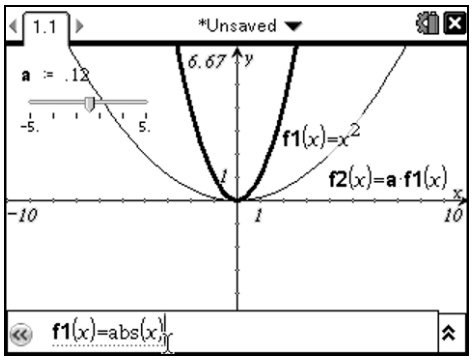
Explore the effect of changing the value of **a** upon the transformation. What happens when **a** is negative?



Extension

Step 1:

You can also change the original function to see what happens for different functions. To do this bring up the input line by pressing ctrl G, and press ▲ until you see *f1(x) =x^2.*
Use del to delete the original function and replace it with any function – for example *abs(x)* is being entered here.



Again use the slider to explore the effect of changing **a** on your new function.

Step 2:

You may also like to change back to *f2(x)=f1(x+a)*, or you could add a second slider and look at:
f2(x)=f1(a·x+b).