Applied Mathematics – a tribute to Leonhard Euler's 315th birthday
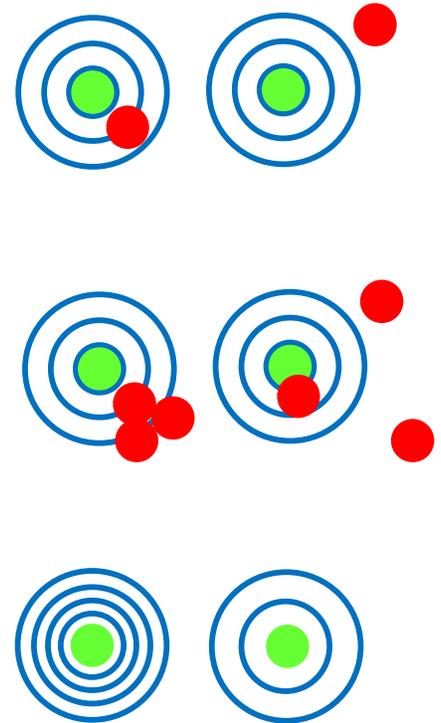
Hans-Martin Hilbig

Measuring angles with the help of gravity

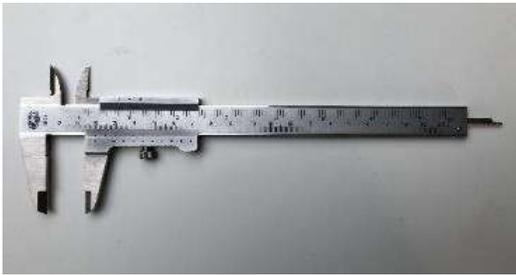# Definitions: Accuracy, Precision, Resolution

- Accuracy:
  How close is a reported measurement to the true value being measured?

- Precision:
  How reproducible is a given measurement?

- Resolution:
  What is the smallest change that can be measured in a given range?

Accuracy, Precision, Resolution

Task:
Measure the length of a stick 150cm long. You have a caliper and a ruler.
Which one is more accurate?

Caliper:

- Resolution: 0.1mm

- Range: 150mm

- Accuracy? Precision?

Ruler:

- Resolution: 1mm

- Range: 2m

- Accuracy? Precision?

Measuring angles with the help of gravity

Spirit level

Technical accuracy of sensor on a perfectly flat surface: 0.029 Degrees

Range of spirit level: +/- 0.3 degrees

Practical accuracy of sensor applied: +/- 0.3 degrees

Quality of electronic sensor measurements

- Sensor properties affecting resolution:
  bit-width, range and **sensitivity** of the sensor

- Methods to affect precision:
  averaging, outlier elimination, noise filtering

- Methods to affect accuracy:
  Mathematical methods, using multiple sources from different sensors

- Sensor sensitivity might not always be constant!!

Measuring angles with an accelerometer
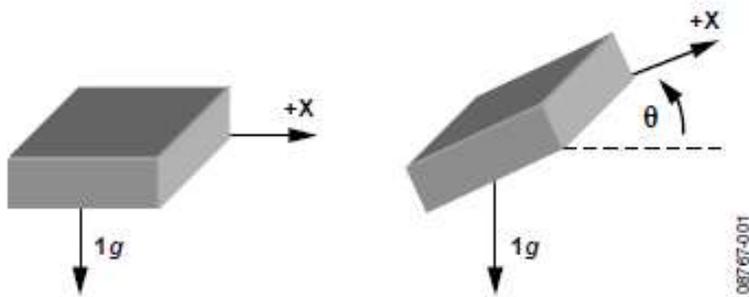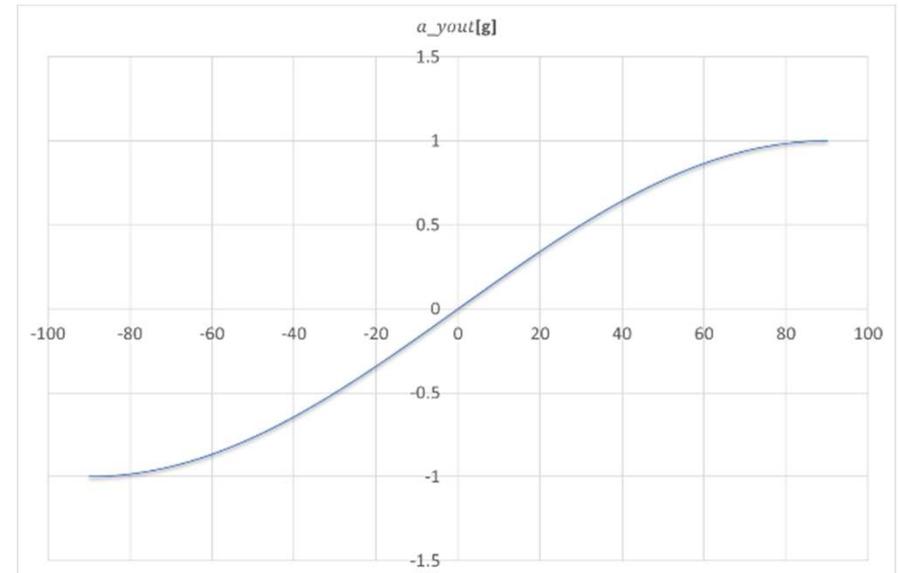
Single x-axis Accelerometer:

Figure 1. Single Axis Used for Tilt Sensing

$$a_{xout} = \sin \theta \times g$$

Challenges:
- Low sensitivity around ϴ = +/- 90 degrees
- Range is limited to +/-90 degrees

Measuring angles with an accelerometer
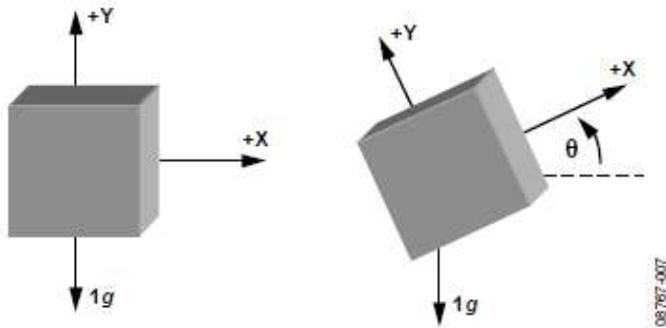
Single y-axis Accelerometer:

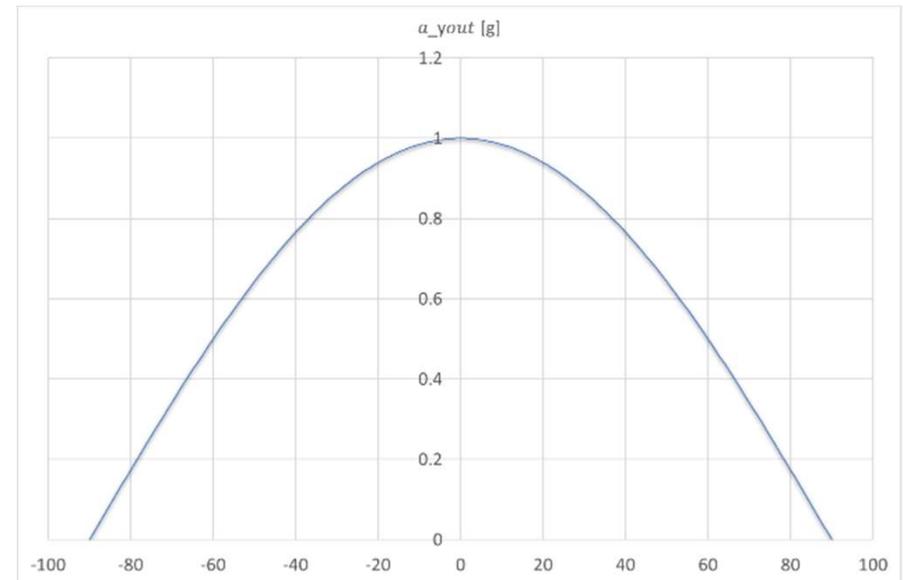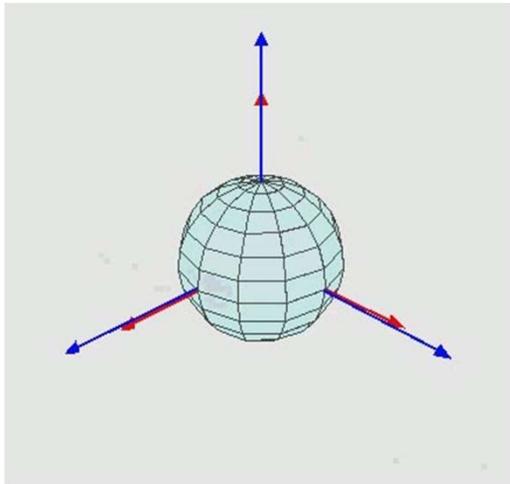*Figure 2: Two axis used for tilt sensing*
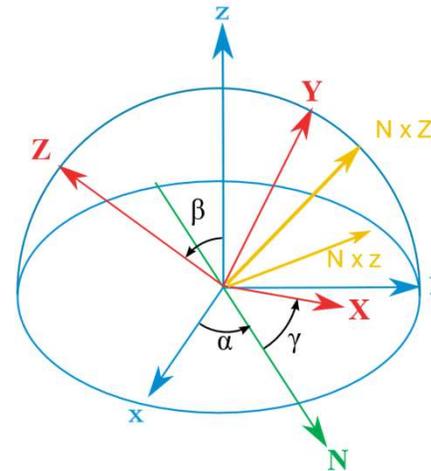
$$a_{yout} = \cos \theta \times g$$

Challenges:
- Low sensitivity around Θ = +/- 0 degrees
- Range is limited to 90 degrees

Euler angles (Wikipedia)

Any target orientation can be reached, starting from a known reference orientation, using a specific sequence of intrinsic rotations, whose magnitudes are the Euler angles of the target orientation.

By Euler2.gif: Juansemperederivative work: Xavax - This file was derived from: Euler2.gif:, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=24338647

By Juansempere - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=6767281

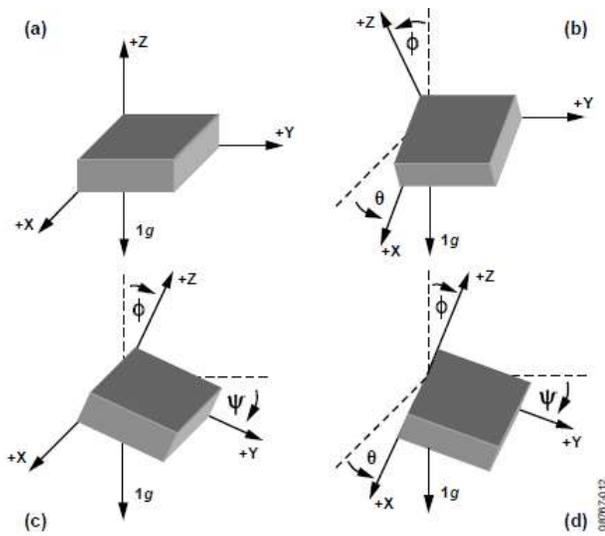# Sensor fusion based on Euler angles

Figure 3: Angles for independent inclination sensing

$$\theta = arctan\left(\frac{a_{xout}}{\sqrt{a^2{}_{yout} + a^2{}_{zout}}}\right)$$

Benefits:
- Sensor fusion compensates areas of low sensitivity
- Quadrant detection extends the range to 0-360 degrees of Theta, Phi and Psi in all three spheres

Summary – key learnings

- Accuracy depends on precision, resolution and sensitivity of sensors

- Mathematical methods and sensor fusion are used to enhance accuracy of an electronic system

- Euler angles applied enhance the single axis accuracy of an electronic system by using a 3-axis accelerometer

- Using an appropriate tool improves accuracy!

Helpful links

- Analog Devices ADXL335 application note AN-1057: Using an Accelerometer for Inclination Sensing:
  https://www.analog.com/en/app-notes/an-1057.html

- T^3 eu, H.M. Hilbig, Veit Berger, Accelerometer library for TI-Innovator under Python:
  https://resources.t3europe.eu/t3europe-home?resource_id=3131&cHash=4457c1aba6400d23c4fb1674b27a2c8f

- … or simply shoot me an email with questions, suggestions, feedback:
  hm-hilbig@web.de

Reference to pictures & graphics used in this presentation:
- Graphic on slide 7: public domain, see: https://commons.wikimedia.org/wiki/File:ConstellationGPS.gif
- Graphics used on slides 8,9,11: ©2010 Analog Devices, Inc. All rights reserved.
  Trademarks and registered trademarks are the property of their respective owners. AN08767-0-2/10(0)
- All other images & graphics: created by the author (Hans-Martin Hilbig 2022)

Sensor fusion to enhance accuracy

GPS:

- 6 x 4 satellites circle the earth twice a day in a precise orbit
- Satellites periodically send their precise & accurate time and position data
- A minimum of four satellites are visible from any given place on earth
- Mathematical algorithms calculate the precise position and velocity on earth

visible sat = 7
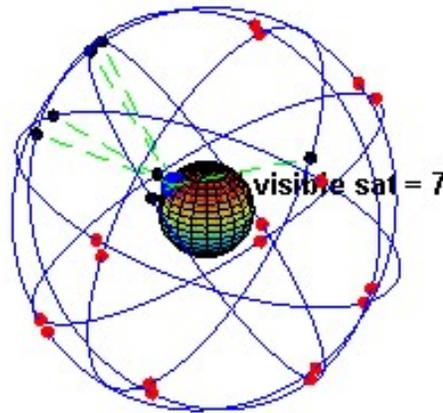
Owners:

- GPS (Navstar): USA
- GLANOSS: Russia
- Galileo: Europe
- BeiDu: China