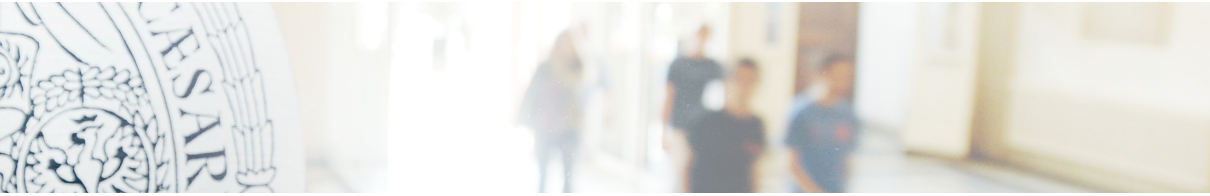
universität innsbruck



Natural number biased?

A technology-aided identification of profiles with respect to rational numbers understanding.

Florian Stampfer

Audience Response Systems (ARS)

- ArsNova (3600 Teacher, 72000 students)
  - Live Feedback
  - Live Assessment
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  - Peer Instruction
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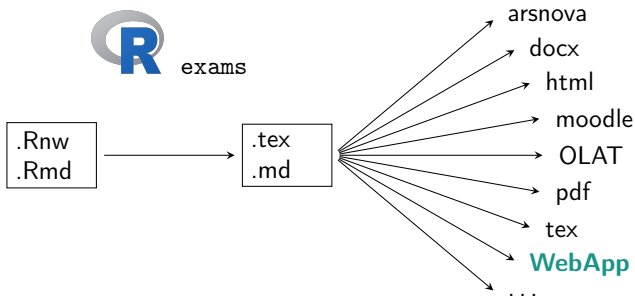
- kahhot!
- . . .

Why it all started . . . (Grün and Zeileis 2009)

- *Scalable exams:* Automatic generation of a large number of different exams in order to provide an individual test to each student.
- *Associated self-study materials:* Collections of exercises and solutions from the same pool of examples.
- *Joint development:* Development and maintenance of a large pool of exercises in a multi-author and cross-platform setting.

$\longrightarrow$ abstract task format . . .

. . . with many export possibilities (Zeileis, Umlauf, and Leisch 2014)



Task Format

- first version: LaTeX-based variant in the form of so-called Sweave files (ending mostly *Rnw* originally **Rn**o**w**eb)
  - *R*-code linked to LaTeX-commands
  - in practice: during the *R* program's runtime, the selected *R* outputs are combined with the LaTeX-commands to form a LaTeX-file

Task Format

- first version: LaTeX-based variant in the form of so-called Sweave files (ending mostly *Rnw* originally **Rn**o**w**eb)
  - *R*-code linked to LaTeX-commands
  - in practice: during the *R* program's runtime, the selected *R* outputs are combined with the LaTeX-commands to form a LaTeX-file

- later versions: additionally a *markdown*-based variant in the form of so-called *R markdown*-files (suffix *Rmd* stands for ***R*** **m**ark**d**own)
  - Markdown code is created during the runtime of the *R* program

Concrete

Structure `Rnw` file:

```
<<echo=FALSE, results=hide>>=
## DATA GENERATION
...
@
\begin{question}
asd
\end{question}
\begin{solution}
asd
\end{solution}

%% META-INFORMATION
...
```

Structure `Rmd` file:

````
```{r,echo=FALSE}
## DATA GENERATION
...

```

Question
========
...

Solution
========
...

META-INFORMATION
================
...
````

A very simple example (1)

```
Question
========
In which month does spring start?

Answerlist
----------
* February
* March
* April
* May

Solution
========
March

META-INFORMATION
================
extype: schoice
exsolution: 0100
```

A very simple example (2)

````
```{r,echo=FALSE}
## DATA GENERATION
questions = c("February","March", "April", "May")
solutions =c(0,1,0,0)
explanation= "March: Spring starts in March since ..."
```

Question
========
In which month does spring start?

```{r, echo=FALSE,results="asis"}
answerlist(questions,markup="markdown")
```

Solution
========
`r explanation`

META-INFORMATION
================
extype: schoice
exsolution: `r mchoice2string(solutions)`
````

Illustrative example - Code

````
```{r,echo=FALSE}
## DATA GENERATION
z <- c(4,9,2,7)
x = z[[1]]/z[[2]];y = z[[3]]/z[[4]]
zahl1=paste0("$\\frac{",z[1],"}{",z[2],"}$");zahl2=paste0("$\\frac{",z[3],"}{",z[4],"}$")
TXT= paste0("Which number is larger: ",zahl1," or ", zahl2," ?")
questions= c(paste0(zahl1," is larger"),paste0(zahl2," is larger"),"Equally large")
solutions=c(x>y,x<y,x==y)
explanation=paste0(questions[solutions],"  since $",z[1],"\\cdot",z[4],"=",z[1]*z[4],
c(">","<","=")[solutions],z[2]*z[3],"=",z[2],"\\cdot",z[3],"$ ist.")
```

Question
========
`r TXT`
```{r, echo=FALSE,results="asis"}
answerlist(questions,markup="markdown")
```

Solution
========
`r explanation`

META-INFORMATION
================
extype: schoice
exsolution: `r mchoice2string(solutions)`
exname: size
````

Illustrative example – pdf

1. **Problem**
   Which number is larger: $\frac{4}{9}$ or $\frac{2}{7}$?

   (a) $\frac{4}{9}$ is larger

   (b) $\frac{2}{7}$ is larger

   (c) equally large

   Solution
   $\frac{4}{9}$ is larger since $4 \cdot 7 = 28 > 18 = 9 \cdot 2$.

Illustrative example – docx

1. **Question**
   Which number is larger: $\frac{4}{9}$ or $\frac{2}{7}$?

   (a) $\frac{4}{9}$ is larger

   (b) $\frac{2}{7}$ is larger

   (c) equally large

   Solution
   $\frac{4}{9}$ is larger since $4 \cdot 7 = 28 > 18 = 9 \cdot 2$.

Illustrative example – html

Exam 1

1. **Question**

   Which number is larger: $\frac{4}{9}$ or $\frac{2}{7}$?

   a. $\frac{4}{9}$ is larger
   b. $\frac{2}{7}$ is larger
   c. equally large

   Solution

   $\frac{4}{9}$ is larger since $4 \cdot 7 = 28 > 18 = 9 \cdot 2$.

Illustrative example – randomization

````
```{r,echo=FALSE}
## DATA GENERATION
z <- c(4,9,2,7)
x = z[[1]]/z[[2]];y = z[[3]]/z[[4]]
zahl1=paste0("$\\frac{",z[1],"}{",z[2],"}$");zahl2=paste0("$\\frac{",z[3],"}{",z[4],"}$")
TXT= paste0("Which number is larger: ",zahl1," or ", zahl2," ?")
questions= c(paste0(zahl1," is larger"),paste0(zahl2," is larger"),"Equally large")
solutions=c(x>y,x<y,x==y)
explanation=paste0(questions[solutions],"  since $",z[1],"\\cdot",z[4],"=",z[1]*z[4],
c(">","<","=")[solutions],z[2]*z[3],"=",z[2],"\\cdot",z[3],"$ ist.")
```

Question
========
`r TXT`
```{r, echo=FALSE,results="asis"}
answerlist(questions,markup="markdown")
```

Solution
========
`r explanation`

META-INFORMATION
================
extype: schoice
exsolution: `r mchoice2string(solutions)`
exname: size
````

Illustrative example – randomization

````
```{r,echo=FALSE}
## DATA GENERATION
z <- sample(1:20,4,replace=TRUE)
x = z[[1]]/z[[2]];y = z[[3]]/z[[4]]
zahl1=paste0("$\\frac{",z[1],"}{",z[2],"}$");zahl2=paste0("$\\frac{",z[3],"}{",z[4],"}$")
TXT= paste0("Which number is larger: ",zahl1," or ", zahl2," ?")
questions= c(paste0(zahl1," is larger"),paste0(zahl2," is larger"),"Equally large")
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```

Question
========
`r TXT`
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answerlist(questions,markup="markdown")
```

Solution
========
`r explanation`

META-INFORMATION
================
extype: schoice
exsolution: `r mchoice2string(solutions)`
exname: size
````

Illustrative example – randomization – RShiny

http://fachdidaktik-mathematik.uibk.ac.at:3838/dimma/mini/?examName=IE-TTT

Task formats (extype)

- num
- schoice/mchoice
- string
- cloze ($\longrightarrow$ exclozetype)

Task formats (extype)

- num: exsolution number (possibly extol)
- schoice/mchoice: exsolution 00010 (mchoice2string)
- string: exsolution string
- cloze ($\longrightarrow$ exclozetype): exsolution exs|exs|...|

Create tasks – exams2xyz

$xyz \in \{pdf, html, pandoc, openolat, arsnova, moodle\} \cup \{dimma\}$

Examples

```
require(exams)
exams2html("illustrativeExample.Rmd")
exams2pdf("illustrativeExample.Rmd",encoding = "utf8")
exams2pandoc("illustrativeExample.Rmd",encoding = "utf8")
```
(default setting is docx)

WebApp

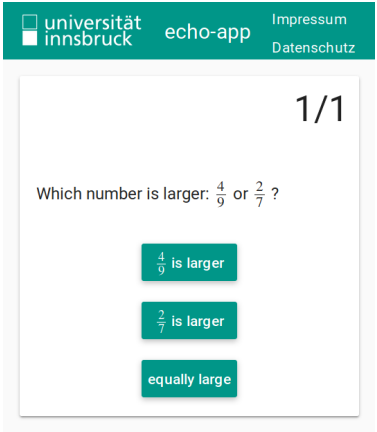
Input

- *exams* tasks

Output

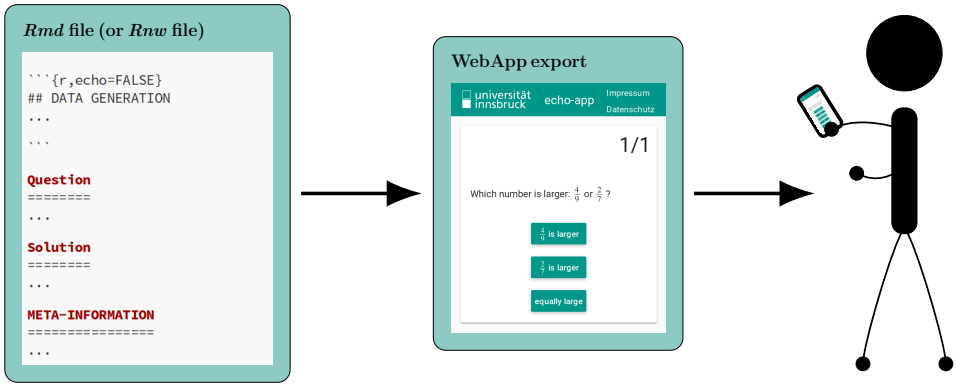
- Data frame (correctness, response time, given answer, . . .)

Illustrative example – WebApp

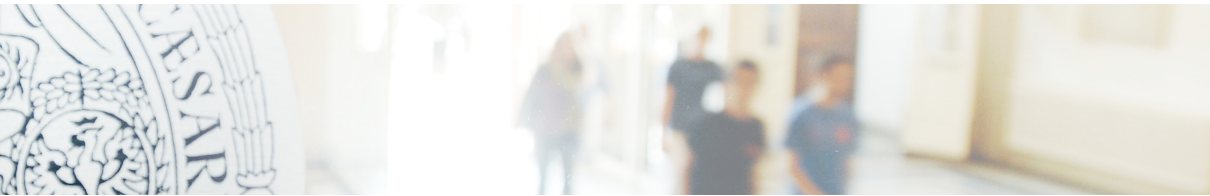


Dummy-Code: SHMONV

Summary Workflow



What comes next . . .



Natural number biased?

A technology-aided identification of profiles with respect to rational numbers understanding.

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What is the *Natural Number Bias* ?

How many numbers lie between 1.9 and 1.40?

Order these numbers: $\frac{5}{6}$, 1, $\frac{1}{4}$, $\frac{4}{3}$.

Which number is missing: $0.36 - 0.2 = \ldots$?

Write down a number between $\frac{1}{4}$ and $\frac{3}{4}$.

Which number is larger: 4.4 and 4.50?

Do you think that $50 \cdot \frac{3}{2}$ is more or less than 50?

What is the *Natural Number Bias* ?

incongruent

How many numbers lie between 1.9 and 1.40?	Order these numbers: $\frac{5}{6}$, 1, $\frac{1}{4}$, $\frac{4}{3}$.	Which number is missing: $0.36 - 0.2 = \ldots$?

congruent

Write down a number between $\frac{1}{4}$ and $\frac{3}{4}$.	Which number is larger: 4.4 and 4.50?	Do you think that $50 \cdot \frac{3}{2}$ is more or less than 50?

What is the *Natural Number Bias* ?

density	size	operations
How many numbers lie between 1.9 and 1.40?	Order these numbers: $\frac{5}{6}$, 1, $\frac{1}{4}$, $\frac{4}{3}$.	Which number is missing: $0.36 - 0.2 = \ldots$?
Write down a number between $\frac{1}{4}$ and $\frac{3}{4}$.	Which number is larger: 4.4 and 4.50?	Do you think that $50 \cdot \frac{3}{2}$ is more or less than 50?

Natural Number Bias

The whole number bias thus refers to a robust tendency to use the single-unit counting scheme to interpret instructional data on fractions.
Ni and Zhou (2005), p, 28

The natural number bias is described as the (inappropriate) application of natural number features in rational number tasks.
Van Hoof, Verschaffel, and Van Dooren (2015), p. 40

Study 1: Western Austria

Study population

- 318 pre-service primary teachers (286 female) of two academic years
- Western Austria (Tyrol and Vorarlberg): Three colleges for teacher education (Pädagogische Hochschulen)
- Before any course on rational numbers
- Conducted in spring 2017 from March to June

Test items

- Rational Number Sense Test: 83 items
- Aspects: 15 *density*, 45 *size*, 23 *operations*
- Congruency: 24 *congruent* and 59 *incongruent* items
- 5 items on demographic data (gender, secondary school types (2), country of graduation, math grade)

Results of the study in Western Austria (1)

- accuracies and response times: clustering mixed-type data

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- accuracies and response times: clustering mixed-type data

group	Strong	NNB	Weak
number	122	125	71

Results of the study in Western Austria (1)

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- dimension reduction: 83 binary variables to 2 dimensions using homogeneity analysis

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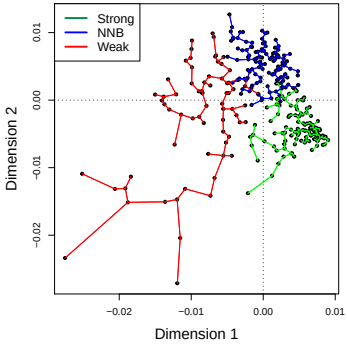
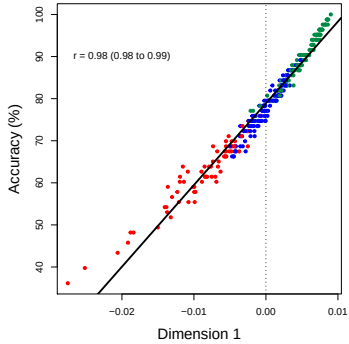
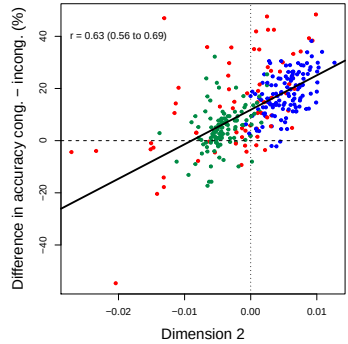
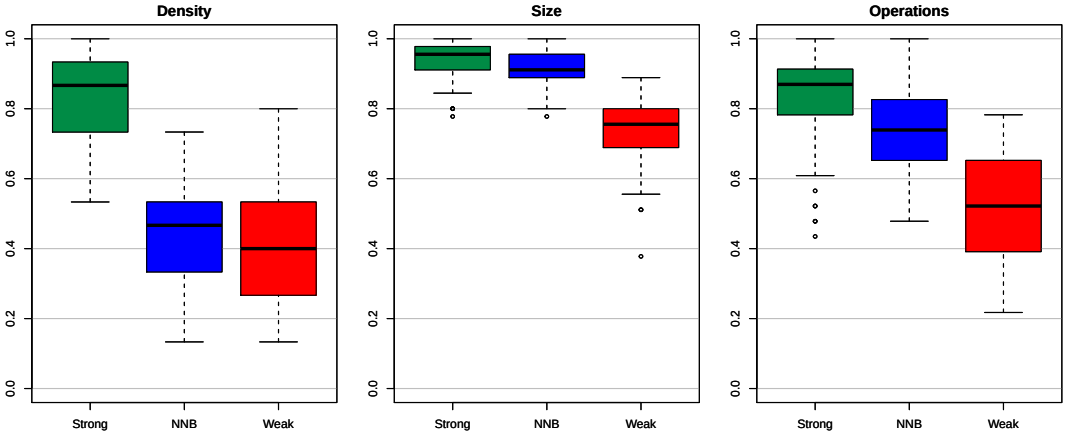






- Dimension 1 ≈ Accuracy
- Dimension 2 ≈ NNB

Results of the study in Western Austria (2)

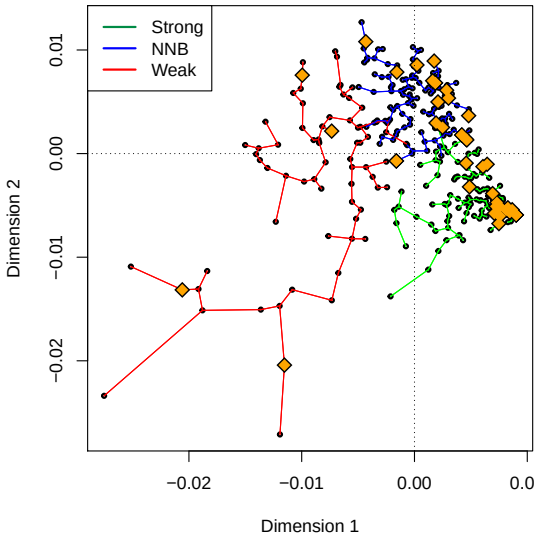


Study 2: University of Potsdam

- Survey (pre/post) by means of WebApp of the students in the teacher training course Mathematics for the Primary School at the University of Potsdam
- Pre-Test: Question set with 83 exercises on the three aspects *density*, *size*, *operations* in the variants congruent and incongruent ; some demographic data; Code for Sample Pairing: 47 (**14**)
- Post-Test: Question set with 46 exercises on the three aspects *density*, *size*, *operations* in the variants congruent and incongruent ; FEMOLA 1[1] with 33 items; Code for sample pairing: 20 (**14**)

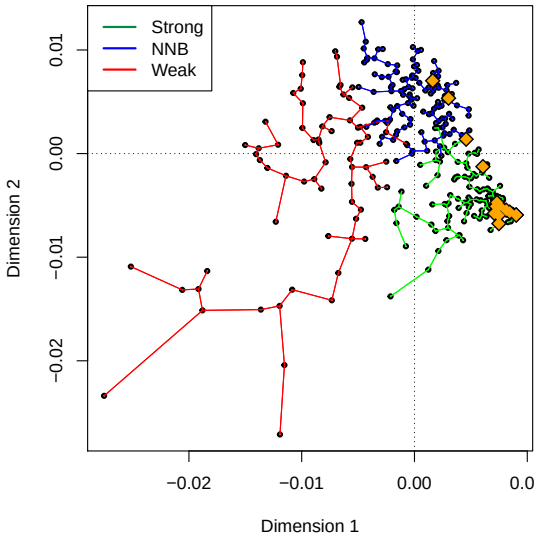
[1] Questionnaire for recording the motivation for the choice of teacher training

Results from Potsdam (M1)



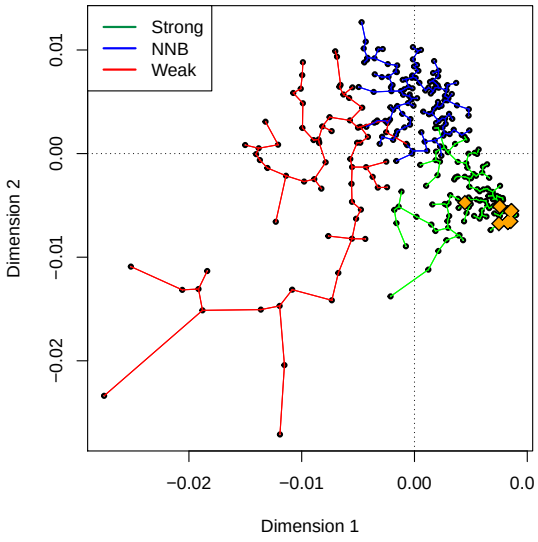
group	Strong	NNB	Weak
number	24	19	4

Results from Potsdam (M1)



group	Strong	NNB	Weak
number	12	2	0

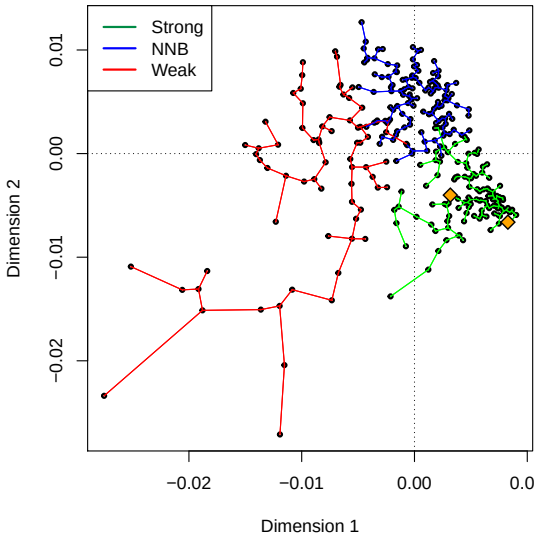
Results from Potsdam (M1+2)



	Strong (M2)	NNB (M2)	Weak (M2)
Strong (M1)	12	0	0
NNB (M1)	2	0	0
Weak (M1)	0	0	0

Results from the talk at the T^3 Sharing Inspiration

Results from the talk at the T^3 Sharing Inspiration



group	Strong	NNB	Weak
number	2	0	0

Discussion

- students in Western Austria show tendencies for NNB
- (also) students in Potsdam show tendencies for NNB
- T^3 Sharing Inspiration participants show tendencies for NNB ?!?

Outlook

- valuable element of the course (orientation for students and lecturer) $\longrightarrow$ adapt lecture
- near future: profile-dependent learning opportunities in technical education
- far future: ...

Literature

Grün, Bettina, and Achim Zeileis. 2009. 'Automatic Generation of Exams in R'. *Journal of Statistical Software* 29 (10): 1–14. https://doi.org/10.18637/jss.v029.i10.

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Zeileis, Achim, Nikolaus Umlauf, and Friedrich Leisch. 2014. 'Flexible Generation of E-Learning Exams in R: Moodle Quizzes, OLAT Assessments, and Beyond'. *Journal of Statistical Software* 58 (1): 1–36. https://doi.org/10.18637/jss.v058.i01.